



Accelerator Physics

Lecture 1 of 2

Rende Steerenberg – CERN



Topics for Today

- The CERN Accelerator Complex
- Main Steps in the LHC Beam Production
- Transverse Motion





Topics for Tomorrow

- Longitudinal Motion
- Some Possible Limitations
- On the Way to HL-LHC...





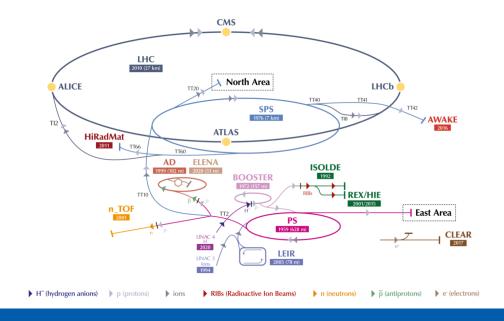
The CERN Accelerator Complex





The LHC and its Injector Complex

- The LHC performance is largely determined by injector complex performance
 - Beam Quality → Beam Brightness
- The PSB determines initial beam brightness
- The PS determines the timing structure
 - 25ns, 50ns, BCMS, 8b4e, ...
- The SPS boosts the energy and creates
 bunch trains

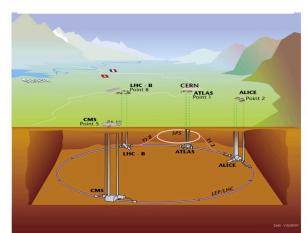


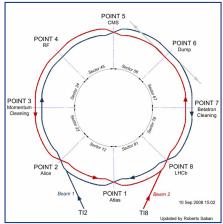




Large Hadron Collider -LHC

- Situated on average ~100 m under ground housing the four major experiments
- Circumference 26.7 km
- Two separate beam pipes going through the same cold mass 19.4 cm apart
- 150 tonnes of liquid helium to keep the magnets cold and superconducting

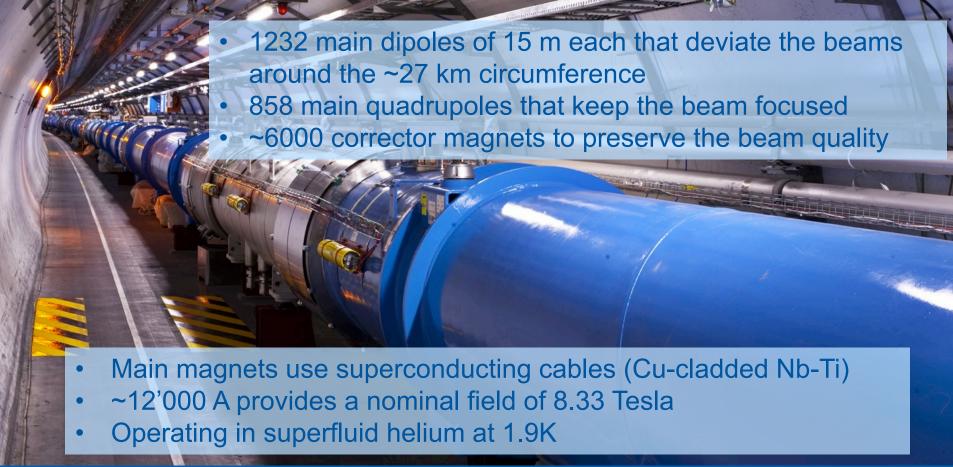










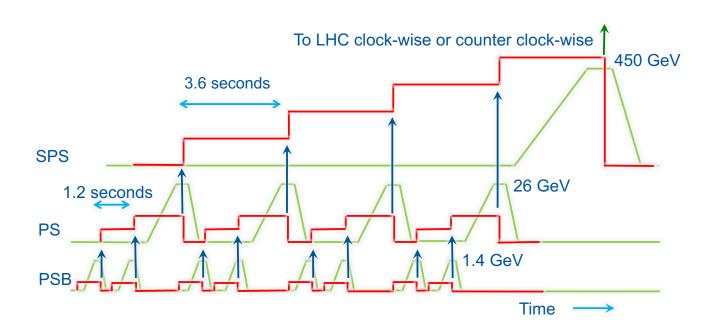






LHC Beam in the Injector Complex

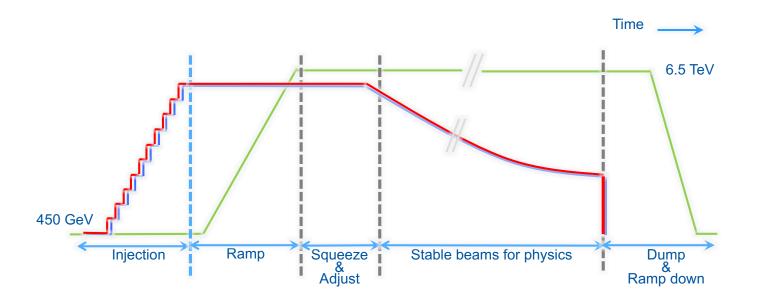
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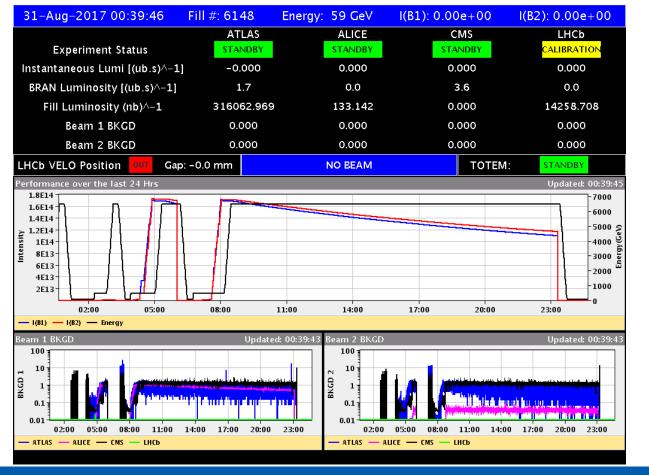


Filling & Cycling the LHC









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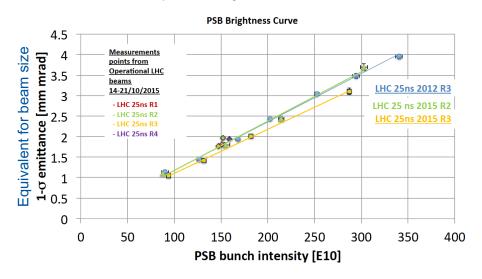
Main Steps in the LHC Beam Production





Injection into the PSB until LS2

With the multi-turn proton injection from Linac 2 the intensity was increased with about constant density





The more hose on the drum the bigger the roll

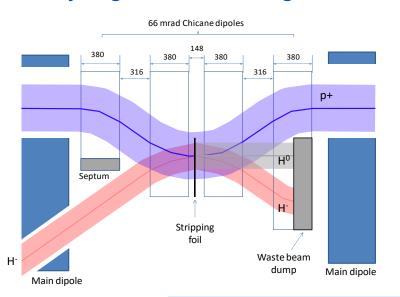
With the new Linac 4 we can increase the density while increasing the intensity



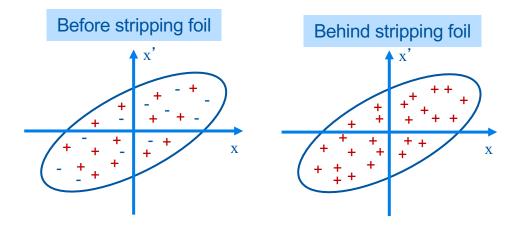


LINAC4 & Charge Exchange Injection

Key ingredient for brighter beams



Use H⁻ beam from LINAC 4



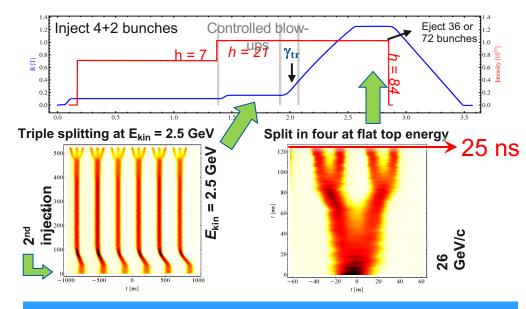
Phase Space Painting is possible (various particle distributions)





Standard LHC 25 ns Beam in PS

- Two injections: 4 + 2 bunches
- Triple splitting at 2.5 GeV
- Two times splitting at 26 GeV/c
- Bunch shortening through bunch rotation before extraction → 4ns bunch length (4σ)
- PSB bunch intensity divided by a factor 12
 - $I_{LHC} = 1.2x10^{11} \text{ ppb} \rightarrow I_{PSB} = 14.4x10^{11} \text{ ppb}$
- Transverse emittance determined by PSB (multiturn injection)
- The SPS creates the bunch trains (288 bunches)



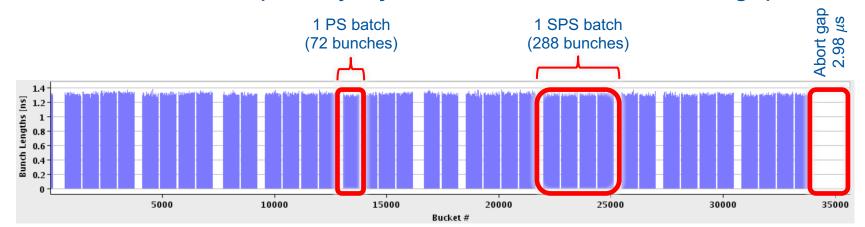
25 ns: Each PSB bunch divided by: $12 \rightarrow 6 \times 3 \times 2 \times 2 = 72$





Create bunch trains in the SPS

- LHC Bunch train structure
- Up to ~ 2800 bunches per beam
- · Bunch trains interrupted by injection and extraction kicker gaps.









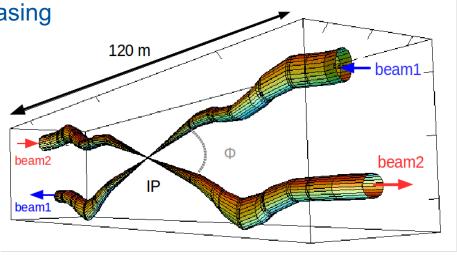
LHC: Squeezing the Beam Size to Collide

Quadrupoles on either side of the experiments

focus the beam to small dimensions, increasing

the local density









Transverse Motion





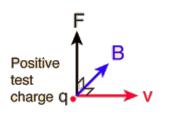
Lorentz Force

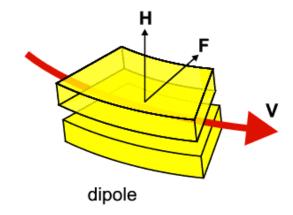
Lorentz Formula:
$$\vec{F} = q\vec{E} + q\vec{v}\vec{x}\vec{B}$$

Magnetic force

The **Transverse motion** is dominated by **magnetic forces**:

$$\vec{F} = q\vec{v} \ x \ \vec{B}$$



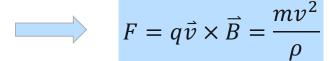






Magnetic Rigidity

The Lorentz Force can be seen as a Centripetal Force



 ρ is the particle's radius of curvature in the magnetic field



$$B\rho = \frac{mv}{q} = \frac{p}{q}$$

• $B\rho$ is the magnetic rigidity

$$B\rho[\text{Tm}] = \frac{mv}{q} = \frac{p[\text{GeV/c}]}{q}$$

$$B\rho = 3.3356 p$$

 Increasing the momentum of a particle beam and keeping the radius constant requires ramping the magnetic fields





Example 1: Radius vs Radius of Curvature

- LHC circumference = 26658.883 m
 - Therefore the radius r = 4242.9 m
- There are 1232 main dipoles to make 360°
 - This means that each dipole deviates the beam by only 0.29°
- The dipole length = 14.3 m
 - The total dipole length is thus 17617.6 m, which occupies 66.09 % of the total circumference
- The bending radius ρ is therefore
 - $\rho = 0.6609 \times 4242.9 \text{ m} \rightarrow \rho = 2804 \text{ m}$
- Apart from dipole magnets there are also straight sections in our collider
 - These are used to house RF cavities, diagnostics equipment, special magnets for injection, extraction and of course the experiments!





Example 2: High Energy LHC

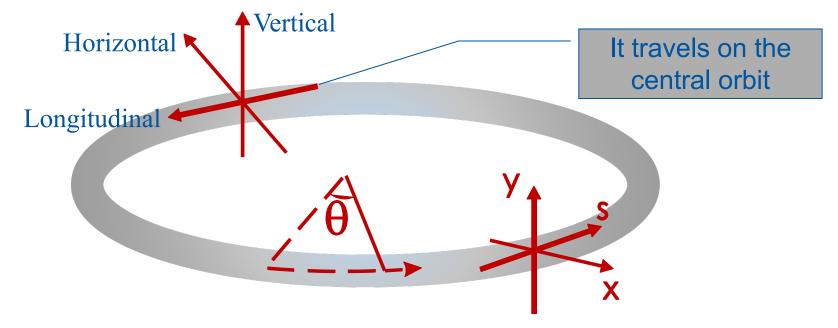
- Use the existing LHC tunnel and replace existing magnets with high field superconducting magnets
- Beam rigidity: $B\rho = 3.3356 p$
- $\rho = 2804 \text{ m}$ (fixed by tunnel geometry and filling factor)
- Vigorous R&D for 20 T dipole magnets is on-going (Nb₃SN and HTS)

$$p = \frac{2804 \times 20}{3.3356} \implies \sim 16.5 \text{ TeV per beam} \implies 33 \text{ TeV}_{cm}$$





Coordinate System Used

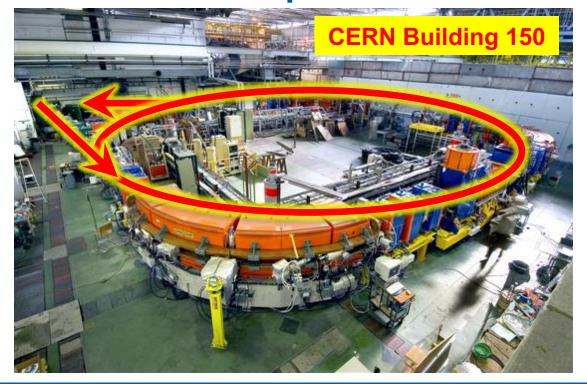


We can speak of a: Rotating Cartesian Co-ordinate System





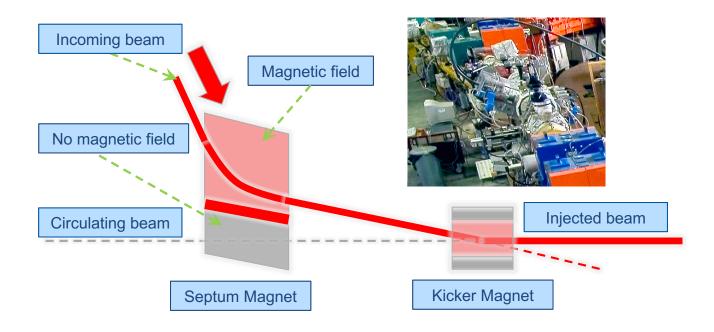
LEIR as an Example







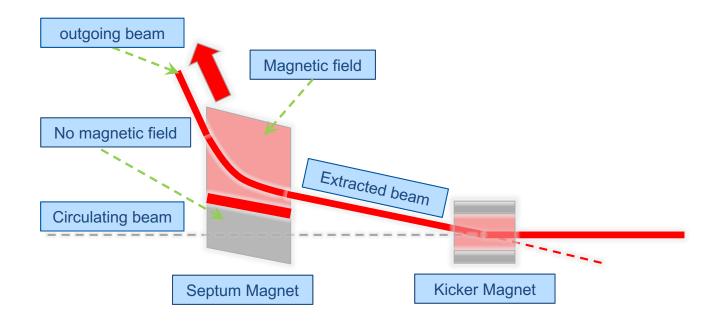
Injecting & Extracting Particles







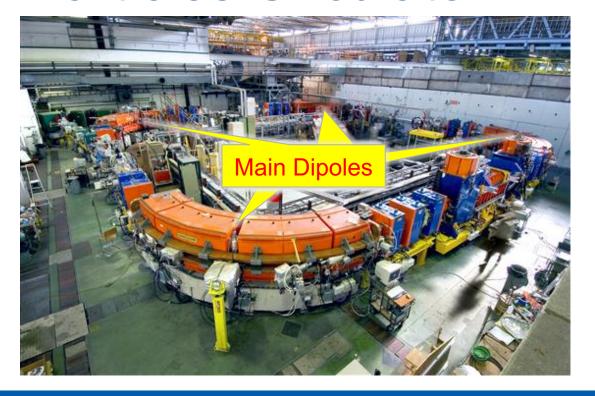
Injecting & **Extracting** Particles







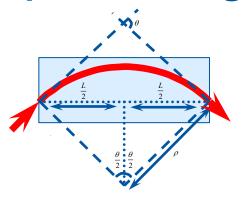
Make Particles Circulate



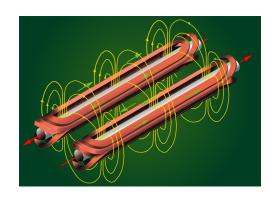




Dipole Magnet







- A magnet with a uniform dipolar field deviates a particle by an angle θ in one plane
- The angle θ depends on the length L and the magnetic field B.

$$\sin\left(\frac{\theta}{2}\right) = \frac{L}{2\rho} = \frac{1}{2} \frac{LB}{(B\rho)}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\theta}{2}$$



$$\theta = \frac{LB}{(B\rho)}$$



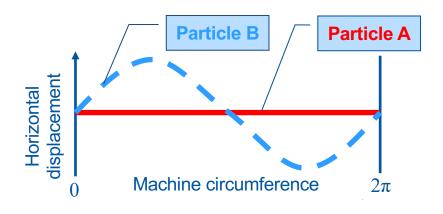


Multiple Particle in a Dipole Field

Two charged Particles in a homogeneous magnetic field

Particle B Particle A

Horizontal motion



Different particles with different initial conditions in a homogeneous magnetic field will follow a closed orbit in the horizontal plane

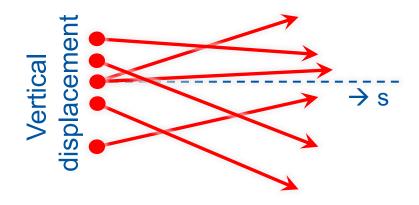




What About the Vertical Plane?

The horizontal motion seems to be "stable".... What about the vertical plane?

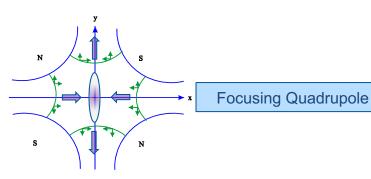
Many particles many initial conditions

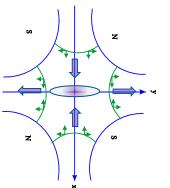




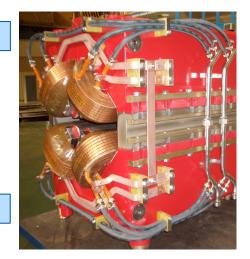


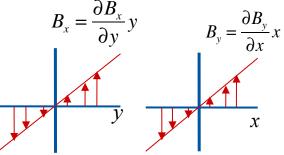
Focussing Particle Beams





De-focusing Quadrupole





Field gradient

$$K = \frac{\partial B_{y}}{\partial x} [Tm^{-1}]$$

$$k = \frac{K}{B\rho} [m^{-2}]$$





FODO Cell

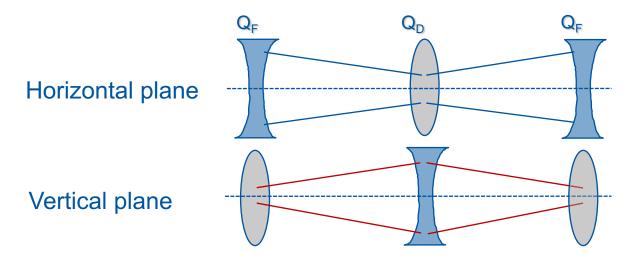
- Using a combination of focusing (Q_F) and defocusing (Q_D) quadrupoles solves our problem of 'unstable' vertical motion.
- It will keep the beams focused in **both planes** when the position in the accelerator,
 type and strength of the quadrupoles are well chosen.
- By now our accelerator is composed of:
 - <u>Dipoles</u>, constrain the beam to some closed path (orbit).
 - Focusing and Defocusing Quadrupoles, provide horizontal and vertical focusing in order to constrain the beam in transverse directions.
- A combination of focusing and defocusing sections that is very often used is the so called: FODO lattice.
- This is a configuration of magnets where focusing and defocusing magnets alternate and are separated by non-focusing drift spaces.





FODO Lattice

A quadrupole is defined focusing if it is oriented to focus in the horizontal plane and defocusing if it defocusses in the horizontal plane



This arrangement gives rise to **Betatron oscillations** within an envelope





Focussing Particle Beams







Hill's Equation

- These betatron oscillations exist in both horizontal and vertical planes.
- The number of betatron oscillations per turn is called the betatron tune and is defined as Q_x and Q_y or also Q_h and Q_v
- Hill's equation describes this motion mathematically

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

- If the restoring force, K is constant in 's' then this is just a Simple Harmonic Motion (Like a pendulum)
- 's' is the longitudinal displacement around the accelerator





General Solutions of Hill's Equation

Position:

$$x(s) = \sqrt{\varepsilon \beta_s} \cos(\varphi(s) + \varphi)$$

Angle:

$$x' = -\alpha \sqrt{\varepsilon/\beta} \cos(\varphi) - \sqrt{\varepsilon/\beta} \sin(\varphi) \varphi$$

- ϵ and φ are constants determined by the initial conditions
- β (s) is the periodic envelope function given by the lattice configuration

$$\varphi(s) = \int_0^s \frac{ds}{\beta(s)}$$

• $\varphi(s)$ Is the phase advance over 1 turn around the machine

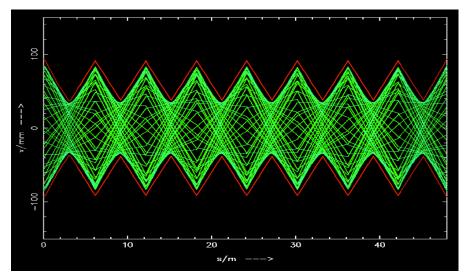
$$Q_{x/y} = \frac{1}{2\pi} \int_0^{2\pi} \frac{ds}{\beta_{x/y}(s)}$$

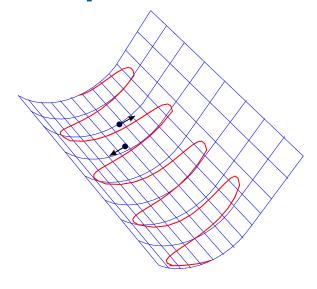
Q_x and Q_y are the horizontal and vertical tunes: the number of oscillations per turn around the machine





β-function and individual particles





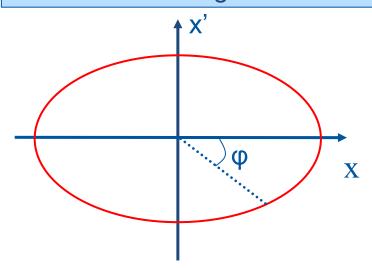
- The β -function is the envelope function within which all particles oscillate
- The shape of the β -function is determined by the lattice

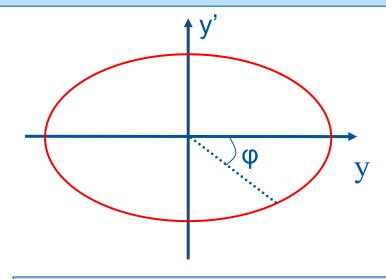




Transverse Phase Space

We distinguish motion in the Horizontal & Vertical Plane





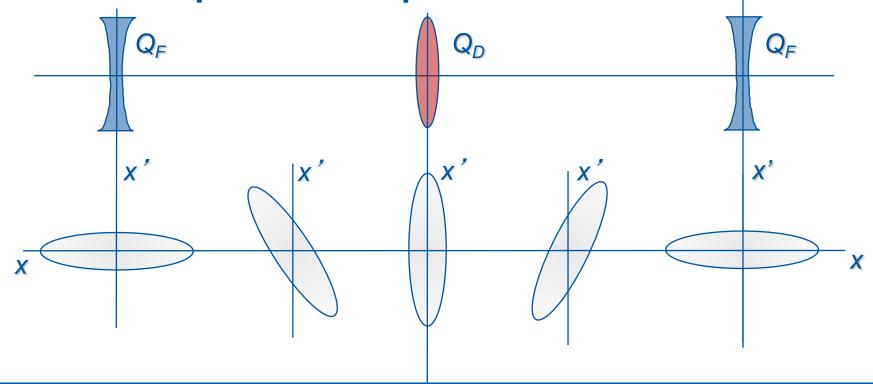
Horizontal Phase Space

Vertical Phase Space





Phase Space Ellipse Rotation

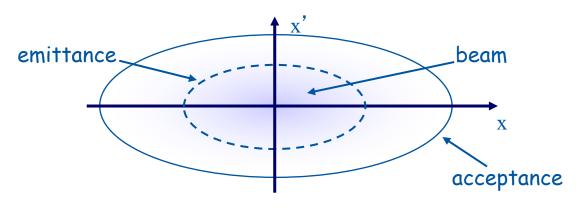






Transverse Emittance

- Observe all the particles at a single position on one turn and measure both their position and angle.
- This will give a large number of points in our phase space plot, each point representing a particle with its co-ordinates x, x'.



Symbol: ε

Expressed in 1σ , 2σ ,...

Units: mm mrad

- The **emittance** is the **area** of the ellipse, which contains all, or a defined percentage, of the particles.
- The <u>acceptance</u> is the maximum <u>area</u> of the ellipse, which the emittance can reach without losing particles





Adiabatic Damping of the Beam Size

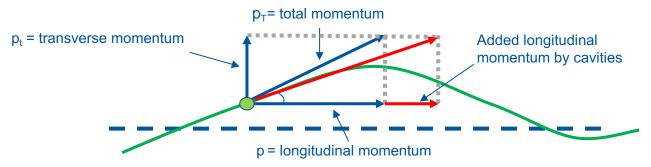
For the Gaussian definition emittance the rms beam sizes are:

$$\sigma_{\chi} = \sqrt{\beta_{\chi} \varepsilon}$$

&

$$\sigma_{y} = \sqrt{\beta_{y} \varepsilon}$$

 The emittance is constant at constant energy, but accelerating particles will decrease the emittance, which is called adiabatic damping



 To be able to compare emittances at different energies it is normalised to become invariant, provided there is no blow up

$$\varepsilon_{x}^{n} = \beta \gamma \varepsilon_{x}$$

$$\varepsilon_y^n = \beta \gamma \varepsilon_y$$





Momentum Compaction Factor

- The change in orbit with the changing momentum means that the average length of the orbit will also depend on the beam momentum.
- This is expressed as the momentum compaction factor, α_p , where:

$$\frac{\Delta r}{r} = \alpha_p \frac{\Delta p}{p}$$

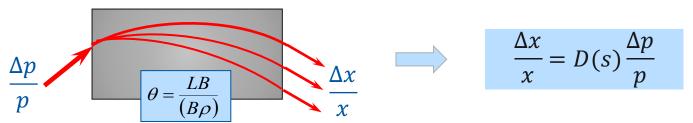
• α_p expresses the change in the radius of the closed orbit as a a function of the change in momentum

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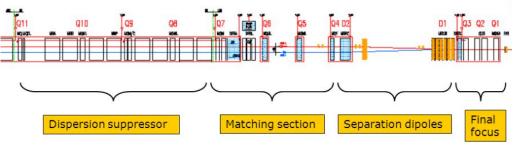


Dispersion

 Our particle beam has a momentum spread that in a homogenous dipole field will translate in a beam position spread at the exit of the magnet



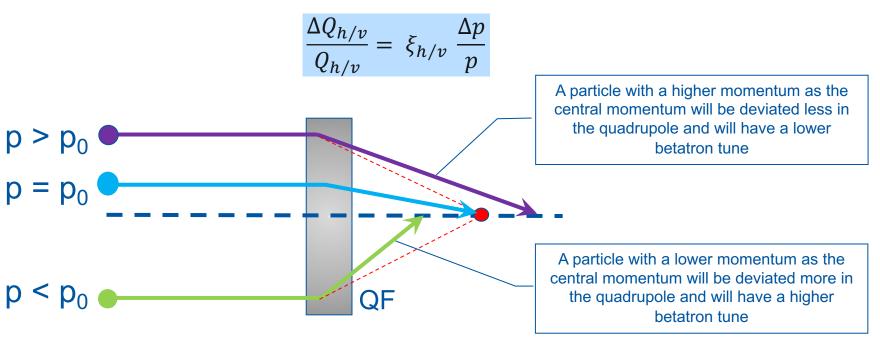
• The beam will have a finite horizontal size due to its momentum spread, unless we install and dispersion suppressor to create dispersion free regions e.g. long straight sections for experiments





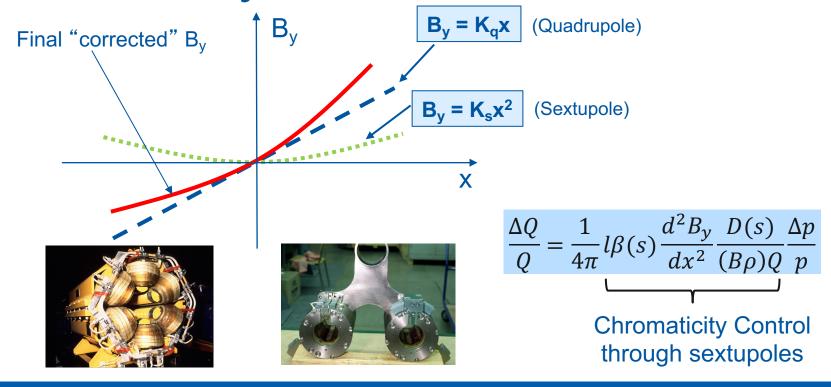
Chromaticity

The chromaticity relates the tune spread of the transverse motion with the momentum spread.





Chromaticity Correction





Tomorrow Morning More....









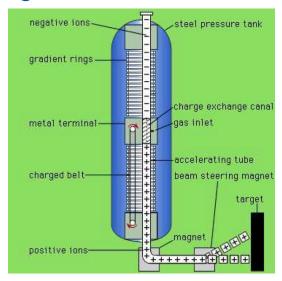
A Very Brief Word on Accelerator History

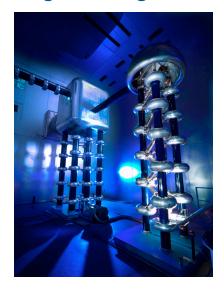




Cockroft & Walton / van de Graaff

- 1932: First accelerator single passage 160 700 keV
- Static voltage accelerator limited by the high voltage needed



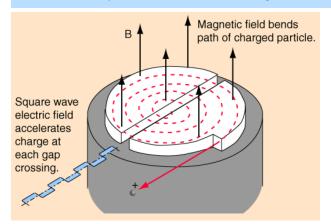


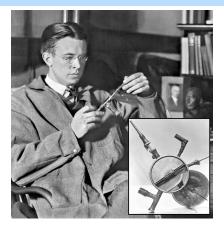




Cyclotron

- 1932: 1.2 MeV 1940: 20 MeV (E.O. Lawrence, M.S. Livingston)
- E = 80 keV for 41 turns
- Constant magnetic field and alternating voltage between the two D's
- Increasing particle orbit radius
- Development lead to the synchro-cyclotron to cope with the relativistic effects (Energy ~ 500 MeV)





In 1939 Lawrence received the Noble prize for his work.



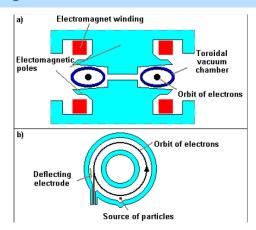


Betatron

- 1940: Kerst 2.3 MeV and very quickly 300 MeV
- First machine to accelerate electrons to energies higher than from electron guns
- It is actually a transformer with a beam of electrons as secondary winding
- The magnetic field is used to bend the electrons in a circle, but also to accelerate them

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A deflecting electrode is used to deflect the particles for extraction.



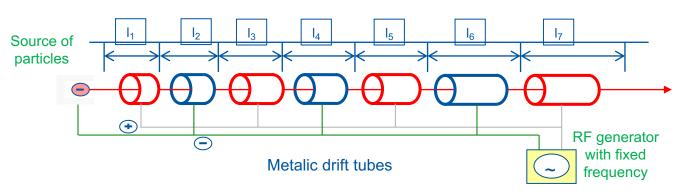




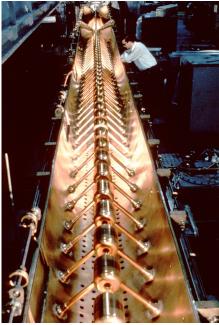


Linear Accelerator

- Many people involved: Wideröe, Sloan, Lawrence, Alvarez,....
- Main development took place between 1931 and 1946.
- Development was also helped by the progress made on high power high frequency power supplies for radar technology.
- Today still the first stage in many accelerator complexes.
- Limited by energy due to length and single pass.



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Synchrotrons

- 1943: M. Oliphant described his synchrotron invention in a memo to the UK Atomic Energy directorate
- 1959: CERN-PS and BNL-AGS
- Fixed radius for particle orbit
- Varying magnetic field and radio frequency
- Phase stability
- Important focusing of particle beams (Courant Snyder)
- Providing beam for fixed target physics
- This invention paved the way to today's colliders

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